Error Modelling for GPS Slant Delays

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A study has been conducted to determine the characteristics of random observation errors in GPS Tropospheric Slant Delays (TSD). An error covariance model has been defined and the construction of the observation error covariance matrix in 3D-VAR has been discussed

1. Introduction

Frequent phase measurements from a dense GPS receiver network are mainly samples made in the same air mass. TSD observations therefore are expected to contain correlated random errors. Information on observation error correlations is required to properly model the observation error covariance matrix R in the observational cost term of a 3D/4D-VAR assimilation system.

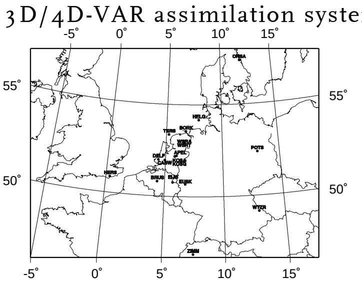

Figure A. *GPS receiver network*

2. Error study

Spatial slant delay error covariances were computed from a 24-day GPS TSD data set of the network in figure A and HIRLAM forecast fields (see [1]). Slant delay errors and model forecast errors are assumed to be uncorrelated. The covariance model

$$\langle \varepsilon^o, \varepsilon^o \rangle = \sigma_i(\alpha_i)\sigma_j(\alpha_j)\rho(r_{ij}) \qquad (1)$$

is used to express observation error covariances as a function of elevation angle and station separation. Figure B gives the results for computing σ at the receiver stations. The dashed line is a fit through all the data and represents the function

$$\sigma(\alpha) = 0.011 + 0.047 \cdot e^{\frac{10-\alpha}{20}} \qquad (2)$$

Figure C gives results for station separation and fits of the model function

$$\rho(r) = \sum_{i=1}^{2} a_i (1 + r/L_i) \cdot e^{(r/L_i)} \qquad (3)$$

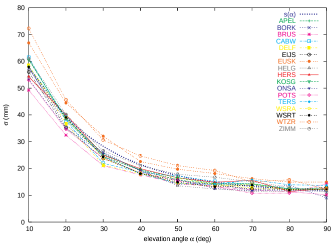

Figure B. *Elevation angle dependence of the observation error standard deviation.*

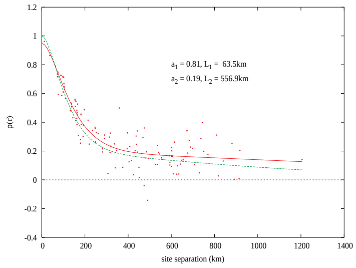

Figure C. *Site separation dependence. Model functions (Bormann[2]) are for unconstrained (red) and constrained fit (green, selected).*

3. Implementation

The construction of the inverse of the positive-definite matrix R_{TSD} in the 3D/4D-VAR cost term for TSDs will be illustrated. Currently temporal correlations in TSDs are discarded which leads to R_{TSD} becoming block-diagonal. For efficiency reasons the following transformation is applied to trivialize the inversion of R_{TSD} in the computation of the cost term (see [1]), i.e.

$$\varsigma = L^{-1}z, R_{tsd} = LL^T \qquad (4)$$

with L being a lower-triangular matrix. For the observations in figure D and their mutual separations in figure E,

stat	lat	lon	azim	elev	date	tim	satid
APEL	52.21	5.96	66.73	58.74	20030502	21:00:00	8
APEL	52.21	5.96	66.20	56.46	20030502	21:05:00	8
BORK	53.56	6.74	206.04	54.50	20030502	21:00:00	10
BORK	53.56	6.74	204.02	52.34	20030502	21:05:00	10
BRUS	50.80	4.36	313.45	16.34	20030502	21:00:00	21
BRUS	50.80	4.36	311.32	16.48	20030502	21:05:00	21
CABW	52.97	4.93	289.85	40.86	20030502	21:00:00	26
CABW	52.97	4.93	290.73	43.06	20030502	21:05:00	26

Figure D. *Observation data.*

	APEL	BORK	BRUS	CABW
APEL	0	136	214	75
BORK	136	0	346	192
BRUS	214	346	0	158
CABW	75	192	158	0

Figure E. *Separation distances (km).*

the different stages of computing R_{TSD} are shown below.

$$R_{TSD} \times 10^3 = \begin{pmatrix} 2.046 & 0.460 & 0.136 & 0.231 & 0 & 0 & 0 & 0 \\ 0.460 & 0.443 & 0.101 & 0.232 & 0 & 0 & 0 & 0 \\ 0.136 & 0.101 & 0.259 & 0.101 & 0 & 0 & 0 & 0 \\ 0.231 & 0.232 & 0.101 & 0.228 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.024 & 0.435 & 0.140 & 0.237 \\ 0 & 0 & 0 & 0 & 0.435 & 0.400 & 0.100 & 0.277 \\ 0 & 0 & 0 & 0 & 0.140 & 0.100 & 0.278 & 0.108 \\ 0 & 0 & 0 & 0 & 0.237 & 0.227 & 0.108 & 0.244 \end{pmatrix}$$

$$L_{TSD} \times 10^3 = \begin{pmatrix} 1.430 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.322 & 0.583 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.095 & 0.121 & 0.485 & 0 & 0 & 0 & 0 & 0 \\ 0.161 & 0.308 & 0.100 & 0.312 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.423 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.306 & 0.554 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.099 & 0.125 & 0.502 & 0 \\ 0 & 0 & 0 & 0 & 0.167 & 0.318 & 0.103 & 0.322 \end{pmatrix}$$

$$L^{-1} \times 10^{-3} = \begin{pmatrix} 0.699 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.386 & 1.716 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.041 & -0.429 & 2.064 & 0 & 0 & 0 & 0 & 0 \\ 0.033 & -1.559 & -0.660 & 3.206 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.703 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.388 & 1.806 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.041 & -0.451 & 1.992 & 0 \\ 0 & 0 & 0 & 0 & 0.033 & -1.641 & -0.637 & 3.104 \end{pmatrix}$$

$$R_{TSD}^{-1} \times 10^{-3} = \begin{pmatrix} 0.640 & -0.696 & 0.136 & 0.105 & 0 & 0 & 0 & 0 \\ -0.696 & 5.560 & 0.144 & -4.999 & 0 & 0 & 0 & 0 \\ -0.106 & 0.144 & 4.694 & -2.116 & 0 & 0 & 0 & 0 \\ 0.105 & -4.999 & -2.116 & 10.281 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.647 & -0.736 & -0.103 & 0.102 \\ 0 & 0 & 0 & 0 & -0.736 & 6.157 & 0.147 & -5.094 \\ 0 & 0 & 0 & 0 & -0.103 & 0.147 & 4.373 & -1.978 \\ 0 & 0 & 0 & 0 & 0.102 & -5.094 & -1.978 & 9.637 \end{pmatrix}$$

Figure F. *Resulting matrices.*

4. Conclusion

An error covariance model for GPS Tropospheric Slant Delays is described and its implementation illustrated.

References

[1] J. de Vries, Progress in error modelling of GPS Slant Delays, Hirlam Newsletter 48, pp 73-81, May 2005.

[2] N. Bormann, S.Saarinen, G. Kelly and J.-N. Thepault, The spatial structure of observation error in the atmospheric motion vectors from geostationary data. Geophysical Research Letters, 27, 2661-2664, 2000

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